Electric Field Enhancement and Power Absorption in Microwave TR-Switches

V. Semenov, M. Lisak, and D. Anderson

Abstract—**An analytical and numerical investigation is made of electric field enhancement due to scattering of an incident plane wave by a biconical conductor. An application to microwave transmit-receive switches (TR) shows that field enhancement factors of the order of 20 to 40 are to be expected in the region close to the keep-alive contacts. An analysis of the microwave absorption by a small plasma sphere located in the vertex of the biconical conductor is also presented, showing that the plasma sphere absorbs a significant fraction of the incident power independently of the plasma size. This explains the observed absorption properties during the turn-on phase of TR switches.**

I. Introduction

IN MANY applications, microwave breakdown in gases plays a beneficial role as in, e.g., microwave transmit-receive switches (TR), [1], [2]. The purpose of the switch is to protect the system by working as a plasma limiter, which allows undisturbed microwave transmission through the TR unit for low powers but blocks out high powers by reflecting against a rapidly self-generated and strongly conducting plasma. In order to obtain short turn-on times of the TR switch, an electron priming source is employed, either in the form of a radio-active material and/or by a small keep alive current through the switch. Furthermore, the priming source is made in the form of sharp truncated cones, which provides the additional advantage of a strong electric-field enhancement (see Fig. 1). This further reduces the breakdown level and contributes to rapid turn-on. The operation of a TR switch involves many characteristic break-down phenomena: the initial break-down of the gas by the ionizing action of the incident wave together with the subsequent, inherently nonlinear interaction between the break-down plasma and microwave. A detailed experimental and theoretical investigation of a number of physical phenomena occurring in TR switches has been presented in [3], where a good agreement between theoretical predictions and experimental results has been found. In a separate investigation [4], the self-consistent and nonlinear interaction between a high-power microwave and a breakdown plasma in TR switches was studied with special emphasis on the properties of power reflection and absorption in a steady state.

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V. Semenov is with the Russian Academy of Sciences, Institute of Applied Physics, Nizhny Novgorod 603600, Russia.

M. Lisak and D. Anderson are with the Institute for Electromagnetic Field Theory and Plasma Physics, Chalmers University of Technology, S-412 96 Göteborg, Sweden.

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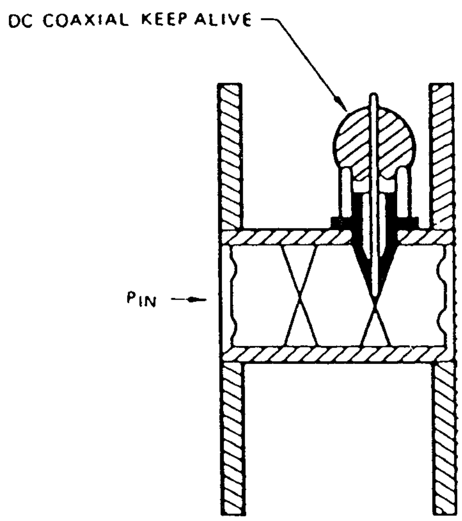

Fig. 1. Typical design of a TR switch.

Although a good agreement between theoretical and experimental results has been obtained in the previous works, the predicted breakdown level in the TR-switch was based on a rough estimate of the average electric field enhancement in the vicinity of the keep-alive contacts. A more detailed knowledge about the field structure close to the contacts is not only necessary for a consistent determination of the breakdown plasma, but also allows the calculation of the power absorped during the creation process of the breakdown plasma. An analysis of these problems is the aim of the present work.

The study of the electric field distribution in a waveguide distorted by the presence of keep alive contacts is a very difficult task since the problem is inherently three-dimensional. Thus, the solutions based upon analytical techniques cannot be obtained in the form of closed expressions and computer-aided numerical analysis becomes imperative. Even the strongly simplified situation where the presence of the waveguide is neglected and only scattering of a plane electromagnetic wave on a perfectly conducting double cone (representing the keep alive contacts) has not been investigated in the literature. The existing investigations have only been concerned with the scattering of a plane electromagnetic wave by a single, perfectly conducting cone (semi-infinite or finite), [5]–[12]. It should be mentioned also that biconical conducting structures have been considered earlier in connection with electromagnetic radiation from a symmetrical, broad band antenna (e.g. [13]–[15]).

Since the main purpose of this work is to determine the electric field enhancement in the vicinity of the contacts, an

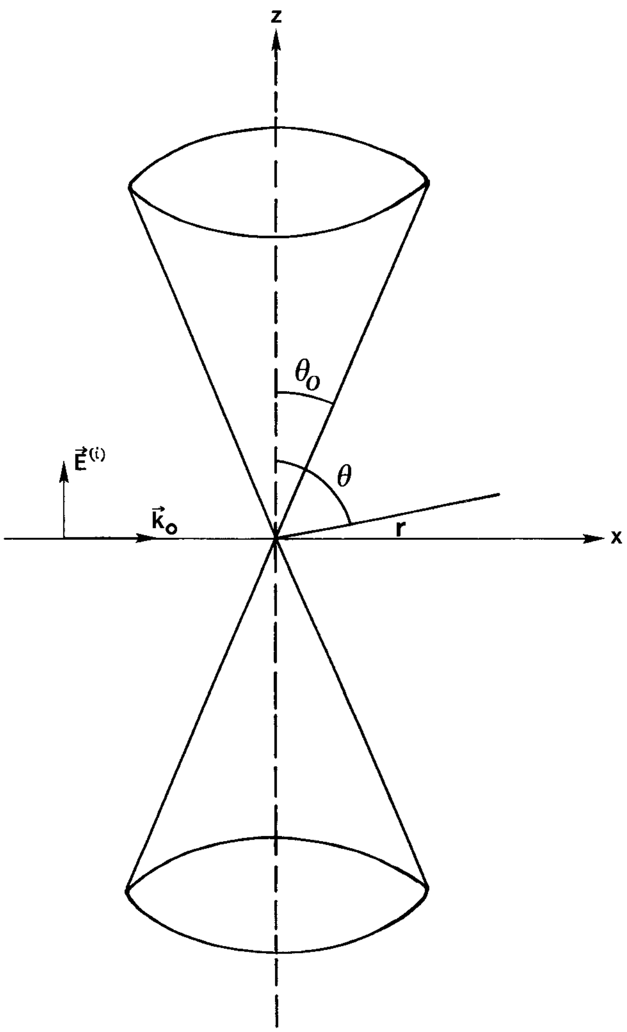

Fig. 2. Idealized geometry of the keep-alive contacts in a TR switch.

analytical approach is applied in Section II to analyze the field structure close to the vertex of the biconical conductor without solving the general scattering problem. In Section III, the electric field distribution and the local electric field enhancement in a realistic TR-switch configuration are investigated numerically by means of the finite element method. The results are in good agreement with those obtained in Section II. In Section IV, the microwave absorption by a small plasma sphere located in the vertex of the biconical conductor is analyzed. It is shown that the plasma sphere absorbs a significant fraction of the incident wave power independently of the plasma size. Finally, Section V summarizes the results and presents conclusions.

II. Electric Field Enhancement Due to Scattering of an Incident Wave by a Biconical Conductor

As our model problem for analytical investigation of the electric field enhancement by the sharp conical keep alive contacts in TR switches, we consider a situation where the geometry of the keep alive contact has been idealized as a double cone according to Fig. 2. A plane wave is incident on the biconical perfect conductor extending from the origin to infinity below and above the x-y plane. The incident wave is polarized in the z-direction. The assumption of an infinite double cone structure is motivated by our interest of the electric field structure close to the vertex of the double cone and it is valid for $k_oL \gg 1$, where $k_o = 2\pi/\lambda$ is the free space wave number and L is the distance between the cavity walls.

Clearly, if one is to attempt a rigorous solution as a boundary value problem, it is appropriate to use spherical coordinates r, θ, φ connected to the (Cartesian coordinates by the usual relations $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$ and $z = r\cos\theta$. Since $\theta = \theta_o$ and $\theta = \pi - \theta_o$ define the surface of the perfect biconical conductor, the boundary conditions for the total field are

$$E_r = E_\varphi = 0 \qquad \text{at } \theta = \theta_o \text{ and } \theta = \pi - \theta_o \tag{1}$$

and for very large values of r, the total field must approach the incident plane wave

$$\vec{E}^{(i)} = E_o(\cos\theta\hat{r} - \sin\theta\hat{\theta})e^{ik_o r\sin\theta\cos\varphi} \tag{2}$$

$$\vec{H}^{(i)} = -\frac{E_o}{Z_o}(\sin\theta\sin\varphi\hat{r}+\cos\theta\sin\varphi\hat{\theta}+\cos\varphi\hat{\varphi})e^{ik_o r\sin\theta\cos\varphi} \tag{3}$$

where $Z_o = \sqrt{\mu_o/\epsilon_o}$ is the intrisic impedance of the free space.

It is well-known (see [16]) that the components of a general field can be expressed in terms of the scalar potentials U and V, which satisfy the Helmholtz wave equation

$$(\nabla^2 + k^2)\,(U, V) = 0 \tag{4}$$

with

$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

Looking for solutions to (4) of the form $U = R(r)G(\theta)\cos(m\varphi)$ and $V = R(r)G(\theta)\sin(m\varphi)$ it may be easily verified that the function $R(r)$ should satisfy the spherical Bessel equation while the function $G(\theta)$ is a solution of the associated Legendre equation.

Because of the nonorthogonality of the spherical Bessel functions on the conductor surface, the potentials U and V for the total field have to be written as series of wave functions with expansion coefficients to be determined. We then obtain the following general solution of the wave equation (4):

$$U = A_o R_o(kr)G_{o,o}(\theta) + \sum_{s=0}^{\infty}\sum_{m=0}^{\infty} A_{v_s}, mR_{v_s,m}G_{v_s,m}(\cos\theta)\cos m\varphi \tag{5}$$

$$V = \frac{1}{Z_o}\sum_{s=0}^{\infty}\sum_{m=1}^{\infty} B_{\mu_s,m}R_{\mu_s,m}G_{v_s,m}(\cos\theta)\sin m\varphi \tag{6}$$

Since the spherical Neumann and Hankel functions are singular at the origin, only the spherical Bessel functions of the first kind may be used, i.e.

$$R_v(kr) = j_v(k_o r) = \sqrt{\frac{\pi}{2kr}}J_{v+\frac{1}{2}}(k_o r) \tag{7}$$

where $J_{v+\frac{1}{2}}(k_o r)$ is the cylindrical Bessel function. The principal mode ($n = 0,\ m = 0$) has been added since the axis ($\theta = 0, \pi$) is not included in the field region. The Legendre function $G_{o,o}(\theta)$ is given by

$$G_{o,o}(\theta) = \ln\left(\operatorname{ctg}\frac{\theta}{2}\right) \tag{8}$$

The boundary conditions (1) are fulfilled if the orders v_s and μ_s are chosen to satisfy

$$G_{v_s,m}(\cos\theta)|_{\theta=\theta_o,\pi-\theta_o} = 0 \tag{9}$$

$$\frac{dG_{\mu_s,m}(\cos\theta)}{d\theta}|_{\theta=\theta_o,\pi-\theta_o} = 0 \tag{10}$$

Equation (9) implies that the associated Legendre functions should be of the form

$$G_{v_s,m\mu_s}(\cos\theta) = \frac{1}{2}\left[P^m_{v_s\mu_s}(\cos\theta) - P^m_{v_s\mu_s}(-\cos\theta)\right] \tag{11}$$

It is easy to show that the $G_{v_s,m}$ functions are orthogonal on the interval $(\theta_o, \pi - \theta_o)$, and that the $G_{\mu_s,m}$ functions form an orthogonal set over the same interval. Equations (9) and (10) determine the eigenvalues ν_s and μ_s, so that for a given value of m only discrete sequences of ν_s and μ_s are possible. In particular, if $m = 0, G_{v_s,0}(\cos\theta)$ has only the first zero, ν_o, below one and all the others exceeding one. If $m > 1$, all values of ν_s and μ_s exceed one. It can be shown that for very small-cone angles, θ_o, the zeros corresponding to the condition (9) are given by [17]:

$$v_s = \begin{cases} 2(m+s) - 1 + \frac{\Gamma(2m+s+1)}{\Gamma(m+1)\Gamma(m)\Gamma(s+1)}\tan^{2m}\frac{\theta_o}{2}; \\ \quad m = 1, 2, \cdots \\ 2s - 1 + \frac{1}{2\ln(2/\theta_o)}; \\ \quad m = 0 \end{cases} \tag{12}$$

where s has all positive integral vales including zero. Now for $r \to 0$, the spherical Bessel function $j_{v_s}(k_o r)$ varies as r^{v_s}. Therefore, it follows from (5) and (6), that the electric field varies as r^{v_s-1} as $r \to 0$, i.e., $E \to 0$ if $\nu_s > 1$ and $E \to \infty$ if $\nu_s < 1$. This means that the main contribution to the field close to the vertex of the double cone corresponds to $\nu_s < 1$, which according to the previous discussion only takes place for the $m = 0$ modes. Consequently, being interested in electric field enhancement near the vertex we neglect the mode with $m > 1$, and only retain two quasi-electrostatic modes:

1) the principal mode $m = 0,\ \nu = 0$ of the form

$$\begin{cases} \vec{E} = -\hat{\theta}A_o\frac{\cos(k_o r)}{r}\frac{1}{\sin\theta}, \\ \vec{H} = -\hat{\varphi}i\frac{A_{p,o}}{Z_o}\frac{\sin(k_o r)}{r}\frac{1}{\sin\theta} \end{cases}, \tag{13}$$

and 2) the mode $m = 0,\ \nu = \nu_o < 1$ given by

$$\begin{cases} \vec{E} = A_{\gamma_o,o}\left[\hat{r}\frac{v_o(v_o+1)}{r}j_{\nu_o}G_{v_o,o} + \hat{\theta}\frac{\partial(rj_{v_o})}{\partial r}\frac{\partial G_{v_o,0}}{\partial\theta}\right] \\ \vec{H} = \hat{\varphi}A_{v_o,0}\frac{ik_o}{Z_o}j_{v_o}\frac{\partial G_{v_o,0}}{\partial\theta} \end{cases} \tag{14}$$

However, the field strength of the mode $m = 0, \nu = \nu_o$ decreases with increasing r and in accordance with the symmetry of the incident field this mode is not excited. In order to determine the excitation of the principal mode, we perform the calculations at where the incident field is undisturbed. Of importance here is the orthogonality relationship

$$\int_{\theta_o}^{\pi-\theta_0}\sin\theta\frac{dG_{v,o}}{d\theta}\frac{dG_{v',o}}{d\theta} = 0 \text{ if } v \neq v' \tag{15}$$

including $\nu = 0$. The matching condition for the θ-components of the incident and total electric fields at $r \to \infty$ is

$$\begin{aligned} &- E_o\int_o^{2\pi}d\varphi\int_{\theta_o}^{\pi-\theta_o}\sin^2\theta e^{ik_o r\sin\theta\cos\varphi}\frac{dG_{o,o}}{d\theta} \\ &= \int_o^{2\pi}d\varphi\int_{\theta_o}^{\pi-\theta_o}E_\theta\sin\theta\frac{dG_{o,o}}{d\theta}d\theta \end{aligned} \tag{16}$$

Since the product of any even mode with any odd mode, or with any mode differing in index m, is obviously orthogonal, we need to consider only such products that do not vanish when integrated over φ from 0 to 2π. Making also use of (15), we obtain from (16) together with (8) and (13)

$$A_o = \frac{E_o}{2\ln\left(\operatorname{ctg}\frac{\theta_o}{2}\right)}\frac{r}{\cos(k_o r)}\int_{\theta_o}^{\pi-\theta_o}d\theta\sin\theta J_o(k_o r\sin\theta), \quad r \to \infty \tag{17}$$

Now applying the asymptotic form of the Bessel function

$$J_o(kr\sin\theta) \sim \sqrt{\frac{2}{\pi k_o r\sin\theta}}\cos\left(k_o r\sin\theta - \frac{\pi}{4}\right), \ r \to \infty \tag{18}$$

the integral in (17) may be evaluated by the method of stationary phase. The stationary point is $\theta = \pi/2$, and the result is

$$\int_{\theta_o}^{\pi-\theta_o}d\theta\sin\theta J_o(k_o r\sin\theta) \sim \frac{2\cos k_o r}{k_o r}, \ r \to \infty \tag{19}$$

which substituted into (17) gives

$$A_o = \frac{E_o}{k_o\ln\left(\operatorname{ctg}\frac{\theta_o}{2}\right)} \tag{20}$$

The electric field close to the vertex of the biconical conductor, i.e., for $kr \ll 1$, is therefore

$$\vec{E} = -\hat{\theta}\frac{E_o}{k_o r\sin\theta}\frac{1}{\ln\left(\operatorname{ctg}\frac{\theta_o}{2}\right)} \tag{21}$$

In order to estimate the local field enhancement in presence of a conical disturbance in the TR-switch, we take as an example the parameters of the experiments presented in [3] i.e., $f = 9$ and 11 GHz, $\theta_o \sim 20°$, $\theta = 90°$, $r \sim 0.09$ mm. The local enhancement factors are then obtained from (21) as $|E/E_o| \sim 27.5$ and 22.5 for frequencies 9 and 11 GHz, respectively. We note, however, that (21) has been derived on the basis of an idealized model and a number of simplifying assumptions. Then, from the point of view of the TR-switch application, a natural question is how well does the above result estimate the electric field enhancement in a realistic TR-switch configuration. To answer this question, a numerical solution of the problem by means of the finite element method is presented in the next section.

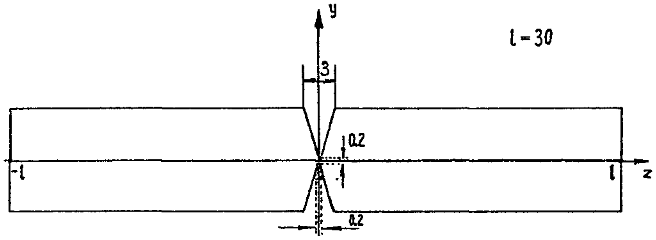

Fig. 3. The geometry of the waveguide with the keep alive contacts used in the numerical calculations (longitudinal cross-section for $x = 0$ plane).

III. Numerical Calculation of the Electric Field Enhancement in Microwave TR-Switches

The numerical analysis of the disturbed microwave electric field in the presence of conical keep alive contacts located in TR-switches is a difficult task since the problem in its character is three dimensional and thus cannot be solved without additional simplifying assumptions. Here, we apply 2 1/2-D analysis by means of the finite-element method. In order to calculate the local electric field enhancement factor, the local field magnitude will be normalized with respect to the incident wave amplitude. This normalization seems to be a justified one since it does not depend on the wave-guide loading.

In the realistic TR-switch configuration considered in [3], the waveguide (2×1 cm) slightly differs from a typical X-band guide, which has the interior dimensions 2.286×1.016 cm. The cut-off frequency of the undisturbed, dominant TE_{10} mode of the waveguide is $f_{c10} = \frac{c}{2b} = 7.5$ GHz ($b = 2$ cm), while the cut-off frequencies of the next higher-order modes become: $f_{c01} = f_{c02} = 15$ GHz, $f_{c11} = 16.77$ GHz. It is evident that frequencies in the X-band (8.2 to 12.4 GHz) can propagate only in the dominant TE_{10} mode. Thus, we represent the incident wave as the TE_{10} mode of the form

$$\hat{E}^{(i)} = \hat{y} E_o \cos\left(\frac{\pi x}{b}\right) \exp\left(i\beta z\right) \tag{22}$$

where

$$\beta = \frac{\omega}{c}\sqrt{1 - f_c^2/f^2}, \; f_c < f. \tag{23}$$

Note that, here, the mode is assumed to propagate along the z-axis. The actual distribution of the electric field in the wave guide without obstacles depends on the possible existence of a reflected wave, which in turn depends on the waveguide loading.

In order to determine the field distribution in the presence of the sharp truncated cones, the finite element method has been applied to a simplified model, since only a two-dimensional analysis FEM program was available. The analysis has been performed for the longitudinal cross-section shown in Fig. 3.

It is assumed that the field distribution along the x-axis is the same as for the incident wave and the x-component of the electric field intensity vector can be neglected. The latter is true for the symmetry plane $x = 0$, but not for other longitudinal cross-sections. The above assumption actually converts the model of the waveguide with cones placed in it into a model in which cones are replaced by wedges. It is evident that such a model cannot be used for the analysis of electromagnetic field transmission through the waveguide, because the reflection from a wedge is much greater than the reflection from a cone. However, the analysis of the local field restricted only to the symmetry plane $x = 0$ should be sufficiently accurate.

In view of the above assumptions we introduce the magnetic Hertz's potential having only one x-component:

$$\vec{\Psi} = \hat{x} \cdot \Psi \tag{24}$$

where the potential Ψ satisfies the wave equation

$$\nabla^2 \Psi + k_o^2 \Psi = 0 \tag{25}$$

and the electric field vector can be evaluated from

$$\vec{E} = i\omega_o \mu_o \nabla \times \vec{\Psi} = i\omega \mu_o \left(\frac{\partial \Psi}{\partial z}\hat{y} - \frac{\partial \Psi}{\partial y}\hat{z}\right) \tag{26}$$

From the assumption that the potential varies with x as

$$\Psi(x, y, z) = V(y, z) \cos\left(\frac{\pi x}{b}\right) \tag{27}$$

the equation for $V(y, z)$ is

$$\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} + \beta^2 V = 0 \tag{28}$$

where $\beta^2 = k_o^2 - (\pi/b)^2$. The components of the electric field vector are then determined by

$$E_y = i\omega\mu_o \cos\left(\frac{\pi x}{b}\right)\frac{\partial V}{\partial z}$$
$$E_z = -i\omega\mu_o \cos\left(\frac{\pi x}{b}\right)\frac{\partial V}{\partial y} \tag{29}$$

The boundary conditions result from the condition that the tantential component of the electric field vanishes at the walls of the waveguide, at the surfaces of the cones and at the symmetry plane $0\ x\ z$. These conditions have the form of the homogeneous Neumann boundary condition:

$$\frac{\partial V}{\partial n} = 0 \tag{30}$$

To complete the formulation of the problem, boundary conditions at the planes $z = \pm 1$ should also be imposed. If these planes are selected sufficiently far from the obstacles, all evanescent modes can be neglected. Therefore, at the input plane $z = -1$ the potential $V(y, -1)$ corresponding to the TE_{10} mode is constant as a function of y. Thus, the boundary condition is a nonhomongeneous Dirichlet one:

$$V = V_o \qquad \text{for} \qquad z = -1 \tag{31}$$

At the output plane $z = 1$ the boundary condition has been established under the assumption that only z-positive travelling TE_{10} wave exists in the region $z > 1$. Hence

$$\frac{\partial V}{\partial z} = i\beta V \qquad \text{for} \qquad z = 1 \tag{32}$$

Equation (28) with the boundary conditions (30), (31), and (32) has been solved by means of the finite element method (isoparametric second order elements have been used). The software package SONMAP has been applied. As a result,

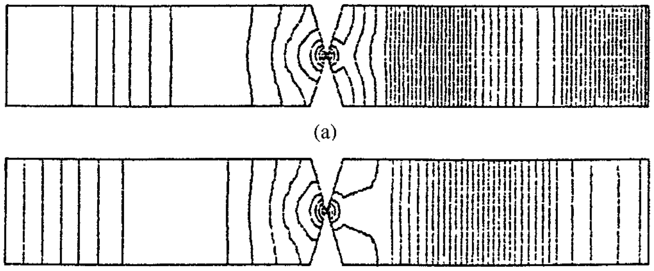

Fig. 4. The electric field patterns for the frequencies. (a) 9 GHz. (b) 11 GHz.

the potential V distribution has been obtained for several frequencies. The equipotential lines have been plotted with the help of the SONMAP graphical postprocessor, and the magnitudes of the electric field density vector have been calculated. The obtained field patterns are shown in Fig. 4(a) and (b). Actually, the electric field lines for $\omega t = \pi/2$ and $x = 0$ have been plotted. It can be seen that the field line structure close to the vertex of the double cone is similar to that of the principal mode that has been considered in the analytical model of the previous section. The local field enhancement factor has been calculated as function of z for three values of y, namely $y = 0$, 0.05; 0.09 mm, and for four frequencies as before. The results are shown in Fig. 5(a) and (b).

It follows that at $y = z = 0.09$ mm the enhancement factors are 31 and 20.5 for frequencies 9 and 11 GHz, in good agreement with the results of the analytical approach of Section II.

IV. Microwave Absorption by a Small Plasma Sphere Located in the Vicinity of the Biconical Contacts

The strong electric-field enhancement considered in the previous sections ensures a lowering of the effective breakdown threshold of the switch. It is obvious that in the initial stage of the breakdown process, the plasma will be created in the region of the strong field, i.e., in the vicinity of the biconical contact. The turn-on, or switching, time as well as the subsequent nonlinear interaction between the plasma and the microwave, depend on the absorption properties of the initial breakdown plasma. According to the experimental observations of [3], the power input into the TR switch at the switching time is an order of magnitude above the peak leakage power. This indicates that most of the incoming power is absorbed in the initial breakdown plasma. It is our purpose here to show that this indication is confirmed by an analytical estimate of the power absorption.

Let us consider a plasma sphere with radius, a, such that $k_o a \ll 1$, and with an arbitrary density distribution. The sphere is located in the center of the perfectly conducting double cone that has been considered in Section II.

The geometry of the wave field incident on the biconical system is also the same as previously. Then, the most significant effect of the interaction between the electromagnetic field and the plasma is connected with the strongest field mode, i.e. the principal mode.

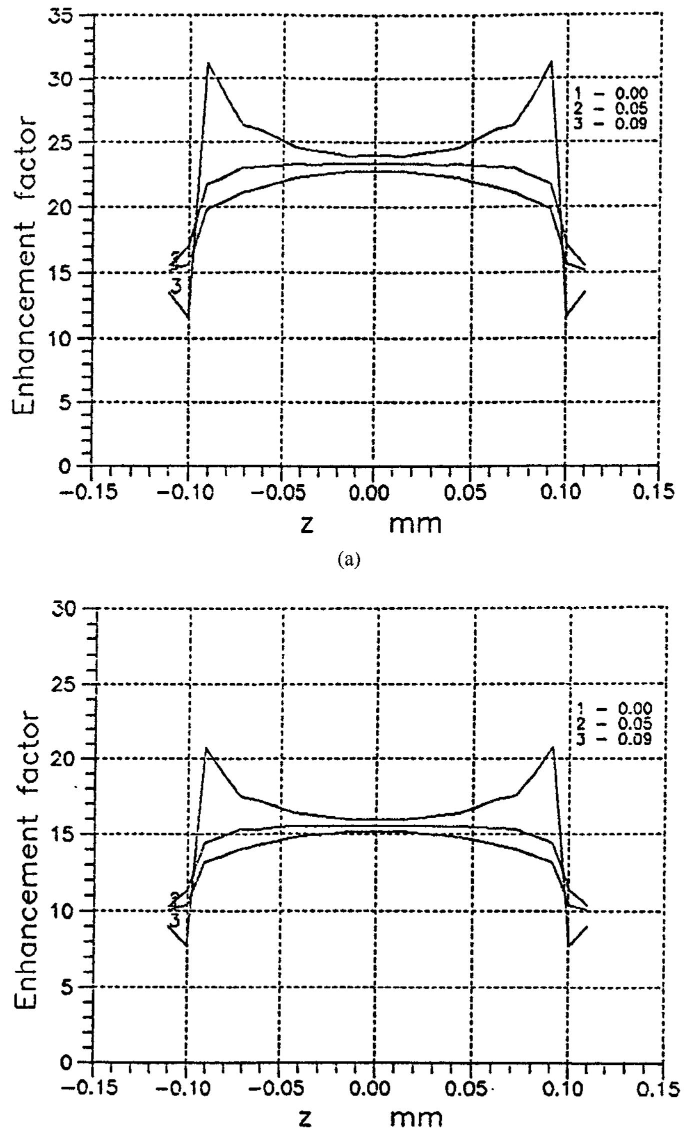

Fig. 5. The enhancement factors as functions of z for the frequencies (a) 9 GHz and (b) 11 GHz.

In order to determine the power absorbed by the plasma, let us first demonstrate the analysis by considering the case of a homogeneous plasma density. In the region outside the plasma $r > a$, the electromagnetic field consists of a wave incident on the plasma and a reflected wave (indicated by subscripts "I" and "R", respectively) which can be represented in the form:

$$\vec{E}_{\substack{I\\R}} = \hat{\theta}\frac{1}{r}\frac{\partial^2(rU_{\substack{I\\R}})}{\partial r\partial\theta}, \tag{33}$$

$$\vec{H}_{\substack{I\\R}} = \hat{\varphi}\frac{ik_o}{Z_o}\frac{\partial U_{\substack{I\\R}}}{\partial\theta}, \tag{34}$$

with

$$U_{\substack{I\\R}} = \frac{1}{2ik_o r}\ln\left(\text{ctg}\frac{\theta}{2}\right) \times \begin{cases} -A_o e^{-ik_o r} \\ Ce^{ik_o r} \end{cases} \tag{35}$$

where A_o and C are constants. The field in the plasma region $r < a$ (denoted by "P") is given by

$$\vec{E}_p = \hat{\theta}\frac{1}{r}\frac{\partial^2(rU_p)}{\partial r\partial\theta}, \tag{36}$$

$$\vec{H}_p = \hat{\varphi}\frac{ik}{Z}\frac{\partial U_p}{\partial\theta} \tag{37}$$

where

$$U_p = Dln\left(ctg\frac{\theta}{2}\right)\frac{\sin kr}{kr} \tag{38}$$

Here, D = const, $k = k_o\sqrt{\varepsilon}, Z = Z_o/\sqrt{\varepsilon}$, and ε is he dielectric constant of the plasma

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega(\omega + iv)} \tag{39}$$

where $\omega_p = (ne^2/m\varepsilon_o)^{1/2}$ is the electron plasma frequency, n is the plasma density, ω is the angular wave frequency, and v is the effective collision frequency.

Matching the tangential components of the field outside and inside the plasma at $r = a$, and using (33)–(38), we obtain

$$C = A_o\frac{\cos ka + i\sqrt{\varepsilon}\sin ka}{\cos ka - i\sqrt{\varepsilon}\sin ka}e^{-2ik_oa} \tag{40}$$

$$D = A_o\frac{e^{-ik_oa}}{\cos ka - i\sqrt{\varepsilon}\sin ka} \tag{41}$$

Note that since $k_oa \ll 1$, the total field in the free space region is only negligibly modified by the small plasma sphere where it approaches the incident plane wave at $r \to \infty$. Consequently, (20) can be assumed to be valid even in the presence of the plasma.

The total power absorption in the plasma, Q, is given by [18]:

$$Q = \frac{\varepsilon_o v\omega_p^2}{2(v^2 + \omega^2)}\int_{V_p}|\vec{E}_p|^2 d^3r \tag{42}$$

where $d^3r = 2\pi r^2\sin\theta dr d\theta$, and V_p is the plasma volume. Using (20) and (41), we obtain from (42)

$$Q = 4\pi\varepsilon_o\frac{\nu\omega_p^2}{(\nu^2 + \omega^2)}\frac{|E_o|^2}{k_o^2\ln\left(\text{ctg}\frac{\theta_o}{2}\right)}\frac{\int_o^a|\cos kr|^2 dr}{|\cos ka - i\sqrt{\varepsilon}\sin ka|^2} \tag{43}$$

Having in mind the assumption $k_oa \ll 1$, we can argue that the largest value of Q is attained at a value of ω_p, which does not violate the condition

$$|ka| = k_oa|\sqrt{\varepsilon}| \ll 1 \tag{44}$$

In this limit (43) becomes

$$Q \cong \frac{4\pi\varepsilon_o\nu Na}{\left[\left(1 + \frac{\nu}{\omega}Nk_oa\right)^2\right]}\frac{E_o}{k_o^2\ln\left(\text{ctg}\frac{\theta_o}{2}\right)} \tag{45}$$

where $N = \omega_p^2/(\omega^2 + \nu^2)$. The maximum absorbed power, $Q_{\max}$, is obtained as

$$Q_{\max} = \frac{2\pi\varepsilon_o\nu}{\left(\frac{\nu}{\omega} + \sqrt{1 + \frac{\nu^2}{\omega^2}}\right)}\frac{E_o^2}{k_o^3 ln\left(ctg\frac{\theta_o}{2}\right)} \tag{46}$$

which occurs at

$$N = N_* = \frac{1}{k_oa\sqrt{1 + (v/\omega)^2}} \tag{47}$$

It is easy to verify that $|\varepsilon| \ll 1$ at $N = N^*$, which assures that (44) is satisfied. Equation (47) shows that a small plasma ball located in the center of the biconical perfect conductor absorbs a finite value of the incident electromagnetic wave power independently of the size of the ball.

Let us now generalize the above results for a plasma with an arbitrary but symmetric density distribution inside the region $r < a$, where $k_oa \ll 1$. Outside the plasma ($r > a$) and in the vicinity of the vertex of the double cone, the field is mainly given by (33)–(35). In the plasma region ($r < a$), we represent the field vectors in the form

$$\vec{E}_p = -\hat{\theta}\frac{1}{\sin\theta}\frac{F(r)}{r}, \tag{48}$$

$$\vec{H}_p = -\hat{\varphi}\frac{1}{\sin\theta}\frac{G(r)}{r}, \tag{49}$$

The functions $F(r)$ and $G(r)$ are determined by the Maxwell equations

$$\nabla \times \vec{E}_p = i\mu_o\omega\vec{H}_p \tag{50}$$

$$\nabla \times \vec{H}_p = -i\omega\varepsilon_o\varepsilon\vec{E}_p \tag{51}$$

which yield

$$\frac{dF}{dr} = i\mu_o\omega G \tag{52}$$

$$\frac{dG}{dr} = i\omega\varepsilon_o\varepsilon F \tag{53}$$

Assuming now that $k_o^2a^2|\varepsilon|_{\max} \ll 1$, where $|\varepsilon^o_{\max}$ is the maximum value of $|\varepsilon|$, we find from (52) and (53) to the first order in $k_o^2a^2|\varepsilon|_{\max}$

$$F(r) \approx F(0), \tag{54}$$

$$G(r) \approx i\omega\varepsilon_oF(0)\int_0^r\varepsilon\, dr \tag{55}$$

From the boundary conditions

$$\begin{aligned} F(r = a + 0) &= F(r = a - 0) \approx F(0), \\ \cdot\, G(r = a + 0) &= G(r = a - 0) \\ &\approx i\omega\varepsilon_oF(0)\int_o^a\varepsilon\, dr, \end{aligned} \tag{56}$$

and from (33)–(35), (48) and (49), we obtain by matching the tangential field components at $r = a$

$$C = A_o \frac{1 + k_o \int_o^a \varepsilon \, dr}{\left(1 - k_o \int_o^a \epsilon \, dr\right)} e^{-2ik_o a}, \tag{57}$$

$$F(0) = A_o \frac{e^{ik_o a}}{1 - k_o \int_o^a \varepsilon \, dr} \tag{58}$$

As follows from (48) together with (54), (58), and (20), the electric field in the plasma is given by

$$\vec{E}_p \approx -\hat{\theta} \frac{E_o}{\ln\left(\mathrm{ctg}\frac{\theta_o}{2}\right)} \frac{e^{ik_o a}}{\left(1 - k_o \int_o^a \varepsilon dr\right)} \frac{1}{k_o r \sin\theta} \tag{59}$$

Substituting (59) into (42) and taking into account that

$$k_o^2 a^2 \int_o^a dr \sqrt{1 + \frac{\nu^2}{\omega^2} N(r)} \ll 1 \tag{60}$$

we obtain the total absorbed power as

$$Q = \frac{4\pi\varepsilon_0 \nu \overline{N} a}{\left[\left(1 + \frac{\nu}{\omega}\overline{N} k_o a\right)^2 + \left(\overline{N} k_o a\right)^2\right]} \frac{E_o^2}{k_o^2 \ln\left(\mathrm{ctg}\frac{\theta_o}{2}\right)} \tag{61}$$

where $\overline{N}a = \int_o^a dr \omega_p^2/(\omega^2 + \nu^2)$, which is exactly the expression (45) with N replaced by $\overline{N}$. Correspondingly, the maximum absorbed power occurs at

$$\overline{N} = \overline{N}^* = \frac{1}{k_o a \sqrt{1 + (\nu/\omega)^2}} \tag{62}$$

and $Q_{\max}$ is given by (46).

In order to estimate what fraction of the input power is being absorbed by the initial breakdown plasma we take the averaged incoming power, $P^{(i)}$, as

$$P^{(i)} \cong \frac{bd}{4Z_o} E_o^2 \tag{63}$$

where b and d are the transverse dimensions of the waveguide. Then, we obtain

$$\frac{Q_{\max}}{P^{(i)}} \cong \frac{8\pi}{bdk_o^2 \ln\left(\mathrm{ctg}\frac{\theta_o}{2}\right)} \frac{1}{\left(1 + \sqrt{1 + \omega^2/\nu^2}\right)} \tag{64}$$

Taking as an example f =10 GHz, θ_o ≅20°, b =2 cm, d =1 cm, and assuming $\nu \cong \omega$, we find from (64) that $Q_{\max}/P^{(i)}$ ∽0.7, which shows that a large fraction of the incoming microwave power can be absorbed in a small region of the initial breakdown plasma. This is clearly in accordance with the experimental observation of [3].

V. Conclusion

The present analysis has considered in detail the problem of determining the averaged electric field enhancement in the vicinity of the keep-alive contact in microwave TR switches. The analytical results derived on the basis of an idealized model of and a number of simplifying assumptions predict field enhancement factors as 27.5 and 22.5 for frequencies 9 and 11 GHz, respectively, which are in good agreement with the results of the numerical analysis. These predictions provide a significant step towards a consistent determination of the breakdown power level as well as the power absorbed in the initial breakdown plasma in TR-switches. An analytical estimate of the microwave power absorption by a small plasma sphere located in the vertex of the biconical conductor shows that the plasma sphere absorbs a large fraction of the incident power, independently of the plasma size. This explains the experimentally observed absorption properties of TR switches during the turn-on phase.

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